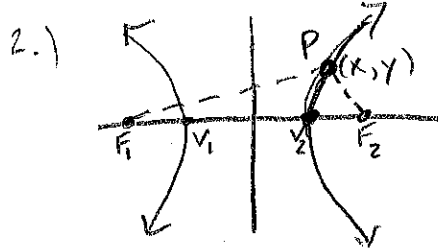


Worksheet 9:4B  
Hyperbolas



$$d(F_1, P) - d(F_2, P) = \pm 2a$$

\* Oblique asymptotes:

• TVA par. to X-AXIS  $y - k = \pm \frac{b}{a}(x - h)$

• TVA par. to Y-AXIS  $y - k = \pm \frac{a}{b}(x - h)$

1. Discuss (find the critical attributes) and graph  $-x^2 + 4y^2 - 2x - 16y + 11 = 0$ .

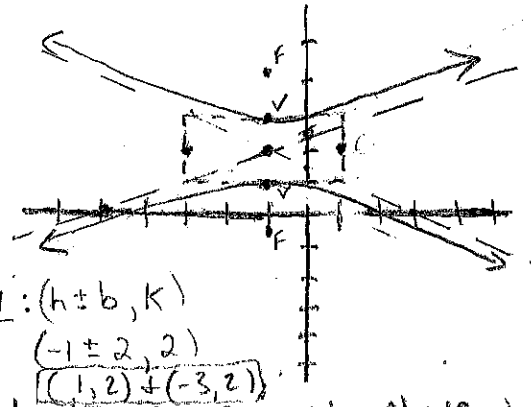
• complete the square for  $x + y$ :

$$-(x^2 + 2x + \boxed{1}) + 4(y^2 - 4y + \boxed{4}) = -11 + -1 + 4(4)$$

$$\frac{-(x+1)^2}{4} + \frac{4(y-2)^2}{4} = \frac{4}{4}$$

$$\frac{-(x+1)^2}{4} + (y-2)^2 = 1 \Rightarrow \frac{(y-2)^2}{1} - \frac{(x+1)^2}{4} = 1$$

• other pts on graph:



• ctr  $(-1, 2)$

• TVA par. to Y-AXIS

•  $a = \sqrt{1} = 1$

•  $b = \sqrt{4} = 2$

•  $c = \sqrt{1+4} = \sqrt{5}$

• Vertices  $(-1, 2 \pm 1)$

• Foci  $(-1, 2 \pm \sqrt{5})$

• Asymptotes:

$$y - k = \pm \frac{a}{b}(x - h)$$

$$y - 2 = \pm \frac{1}{2}(x + 1)$$

• Co-V:  $(h \pm b, k)$

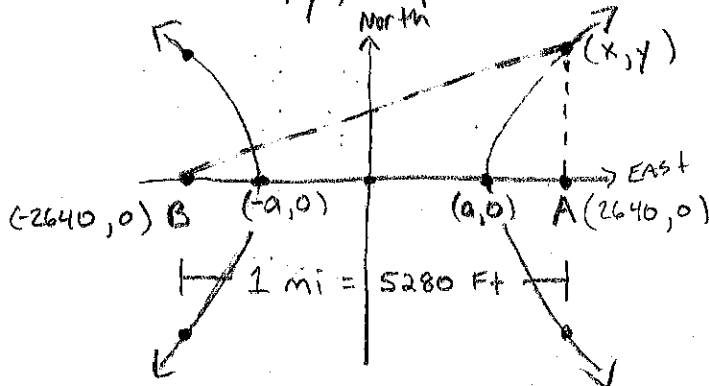
$$(-1 \pm 2, 2)$$

$$(-1, 2) + (-3, 2)$$

\* Since TVA is par. to Y-AXIS, the vertices + foci are located  $a \pm c$  units above & below the center.

2. Suppose that two people standing 1 mile apart both see a flash of lightning. After a period of time, the person standing at point A hears the thunder. One second later, the person standing at point B hears the thunder. If the person at B is due west of the person at A and the lightning strike is known to occur due north of the person standing at point A, where did the lightning strike? (Sound travels at 1100 feet per second)

• Let  $(x, y)$  represent the location of the lightning strike.



• Since sound travels at 1100 Ft/sec + the person standing at point B hears the thunder 1 sec. later than the person at point A, the difference of the distance from  $(x, y)$  to A and from  $(x, y)$  to B is  $2a$  [which equals 1100].

$$2a = 1100 \Rightarrow a = 550$$

•  $2c =$  distance b/w the foci

$$2c = 5280 \Rightarrow c = 2640$$

$$b^2 = c^2 - a^2 \Rightarrow b = \sqrt{2640^2 - 550^2} = 2582.073 = b$$

• Since lightning strikes due north of A, the coordinates for  $(x, y)$  are  $(2640, y)$ . Thus sub 2640 in for  $x$  +

Solve for  $y \rightarrow \frac{2640^2}{550^2} - \frac{y^2}{2582.073^2} = 1 \Rightarrow y = 12,122 \text{ Ft}$

• TVA is parallel to X-AXIS, so

use  $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

$$\frac{x^2}{550^2} - \frac{y^2}{2582.073^2} = 1$$

$$\frac{x^2}{302,500} - \frac{y^2}{6,667,100} = 1$$